Cross-borehole resistivity tomography of sea ice

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Abstract

The presence of brine inclusions with an alignment that is preferentially vertical means that the bulk resistivity structure of sea ice is anisotropic. This complicates the interpretation of surface resistivity soundings of sea ice. We show that consideration of the theory of resistivity measurements in an anisotropic medium suggests that cross-borehole measurements using one current and one potential electrode in each borehole should allow the determination of the horizontal component of the anisotropic bulk resistivity. A series of cross-borehole measurements made in first-year sea ice near Barrow, Alaska, as the ice warmed through the spring, yields 3D models of the resistivity structure which support this prediction. The derived models show an evolution of the resistivity structure which (1) at temperatures less than −5 °C is broadly consistent with the expected variation with brine volume fraction predicted by Archie’s Law and (2) shows evidence of a percolation transition in the horizontal component of the resistivity when brine volume fractions exceed 8–10%.

Keywords: Sea ice; Electrical resistivity; Tomography

1. Introduction

Pockets of brine trapped within sea ice have a determining influence on its physical (Trodahl et al., 2001; Eicken, 2003) and biological (Krembs et al., 2000) properties, and, through these, on global climate (Saenko et al., 2002; Beckmann and Goosse, 2003) and ecology. In particular the bulk properties of sea ice are sensitive to the degree of connectivity of the brine pockets within the ice-brine composite. The connectivity permits the flow of brine and, in turn, of nutrients and heat. Thus, for example, the heat flow in sea ice shows anomalies associated with brine and meltwater movement (Lytle and Ackley, 1996; Pringle et al., 2007).

In theory the internal structure of sea ice can be studied using any transport property to which the brine and ice components contribute differently. Fluid transport, which requires a direct measurement of brine permeability, has been studied by several authors using NMR techniques (e.g. Callaghan et al., 1998, 1999; Eicken et al., 2000; Mercier et al., 2005) as well as field-based methods (e.g. Freitag and Eicken, 2003). However, there are severe difficulties in making accurate measurements under the required in situ constraint. An alternative method to try and resolve the internal structure of sea ice utilizes the electrical resistivity of the ice, which can, in principle, be measured using the direct current (dc) resistivity geophysical technique. DC
resistivity soundings, made on the surface of the sea ice, have previously been discussed by Fujino and Suzuki (1963), Thyssen et al. (1974), Timco (1979) and Buckley et al. (1986). Much of this work focussed on the ability of resistivity measurements to determine the total ice thickness, although Timco (1979) attempted to relate the measured resistivity to the microstructure of the ice by developing a model for the way in which the bulk resistivity depended upon the geometry of brine inclusions. However, the observed preferential vertical orientation of brine pockets leads to the bulk resistivity being anisotropic and this causes significant complications in the interpretation of surface based resistivity data.

In this paper we briefly review the theory behind resistivity measurements in a medium in which the bulk resistivity is anisotropic. We use this to show that, in contrast to surface measurements of resistivity, which are sensitive only to the geometric mean of the principal resistivities, cross-borehole resistivity measurements should be able to not only determine an accurate thickness of sea ice, but also yield measurements of the horizontal component of the bulk resistivity. We demonstrate this by presenting the results of a series of field measurements made on first-year sea ice at Barrow, Alaska over the period during which the sea ice underwent substantial warming during the spring.

2. Electrical resistivity measurements in an anisotropic medium

The preferential vertical elongation of brine inclusions in first-year sea ice means that the bulk electrical resistivity structure is anisotropic, with the resistivity in the vertical orientation \( \rho_V \) being lower than that \( \rho_H \) in horizontal directions. The treatment of electrical resistivity measurements in an anisotropic medium has been discussed in detail by Bhattacharya and Patra (1968). The principal results (see Appendix) are expressed by Eqs. (1a) and (1b)

\[
V = \frac{I\rho_m}{4\pi R\left\{1 + (\lambda^2 - 1)\cos^2\theta\right\}^{1/2}} \quad (1a)
\]

and

\[
V = \frac{I\rho_m}{2\pi R\left\{1 + (\lambda^2 - 1)\cos^2\theta\right\}^{1/2}} \quad (1b)
\]

These expressions give the electric potential \( V \) observed at a location (distance \( R \), polar angle \( \theta \)) from an electrode through which current is injected into and taken out of the subsurface through the two outer electrodes and the resulting potential difference \( \Delta V \) is measured between the other two electrodes. If the electrodes are equally spaced, with a separation \( a \) between adjacent electrodes (the Wenner configuration), the potential difference which would be measured over a uniform anisotropic medium can be derived from (1b) to be

\[
\Delta V = \frac{I\rho_m}{2\pi a} \quad (2)
\]
As bulk resistivity generally varies with depth the measured value of $\Delta V$ for a particular current and electrode separation is used to calculate an apparent resistivity $\rho_a$ defined by

$$\rho_a = 2\pi a \frac{\Delta V}{I}. \quad (3)$$

In a resistivity sounding a series of measurements is made with the separation of electrodes increased in increments to obtain the variation of apparent resistivity with $a$. In the situation of isotropic bulk resistivity this variation may be interpreted in terms of the variation with depth of the bulk resistivity. However, it is clear from Eq. (2) that over a medium with anisotropic resistivity, resistivity soundings are sensitive to the geometric mean resistivity $\rho_m$. As sea ice is underlain by saltwater which has a resistivity of $\sim 0.3 \, \Omega\text{m}$, significantly lower than that generally observed for $\rho_m$, resistivity soundings might also be expected to yield a method of determining ice thickness. However, in practice, as has been observed by several authors (e.g. Buckley et al., 1986; Thyssen et al., 1974; Timco, 1979), the ice thickness is underestimated by such methods. For a uniform thickness of sea ice (i.e. $\rho_V$ and $\rho_H$ constant throughout) the true thickness $t$ is underestimated by the factor $\lambda$. As an example of this, Fig. 2 shows the interpretation of a Wenner sounding made on 1.45 m thick first-year sea ice at Barrow, Alaska. Two derived resistivity models (dashed/dotted lines and layered structure) are shown. These represent the models with the maximum and minimum values of resistivity for the second layer which are able to fit the data. Both models suggest a thickness of sea ice, overlying the low resistivity sea-water, which is substantially less than the true thickness—indicating a mean value of $\lambda$ of 0.1–0.25. This is somewhat less than the values of $\lambda$ found by Timco (1979) at Pond Inlet, N.W.T. or Buckley et al. (1986) in the Antarctic. The inferred mean resistivity of the sea ice is also shown not to be constant—in this example the sea ice is modelled in terms of 2 layers with different mean resistivity values. Notwithstanding the non-uniqueness of the interpretation, shown by the differences in models that provide similar fits to the data, the anisotropy of the resistivity structure can clearly be seen to further complicate the interpretation of resistivity soundings made on sea ice.

An alternative method of measuring the resistivity structure of sea ice is provided by the less common technique of cross-borehole resistivity tomography. In its simplest form, as shown in Fig. 1b, this technique uses vertical arrays of electrodes embedded in a medium. As in resistivity sounding, current is injected into and taken from the medium through 2 electrodes and the resulting potential difference measured between two others. Cross-borehole resistivity tomography has been used in a number of shallow engineering and hydrological studies (e.g. Daily et al., 1992; Deceuster et al., 2006; Ramirez et al., 1996; Slater et al., 1997; Turner and Acworth, 2004). To assess the potential of this type of measurement to investigate the internal resistivity structure of sea ice consider the situation where the current electrodes are located at $(x_1, z_1)$ and $(x_2, z_2)$, and the potential difference is measured between electrodes at $(x_1, z_3)$.
and \((x_2, z_4)\). Thus one current and one potential electrode is in each of 2 boreholes. In this situation application of Eq. (1a) leads to an expression for the measured potential difference

\[
\Delta V = \frac{I}{4\pi} \frac{\rho_m}{\lambda} \left\{ T_1 + T_2 + T_3 + T_4 \right\}
\]

(4)

where

\[
T_1 = \frac{1}{|z_3 - z_1|}
\]

(5a)

\[
T_2 = \frac{1}{|z_4 - z_2|}
\]

(5b)

\[
T_3 = -\frac{\lambda}{\left\{ (x_2 - x_1)^2 + (z_3 - z_2)^2 \right\}^{1/2}} \left\{ 1 + (\lambda^2 - 1)\sin^2 \left( \tan^{-1} \frac{z_3 - z_2}{x_2 - x_1} \right) \right\}^{1/2}
\]

(5c)

and

\[
T_4 = -\frac{\lambda}{\left\{ (x_2 - x_1)^2 + (z_4 - z_1)^2 \right\}^{1/2}} \left\{ 1 + (\lambda^2 - 1)\sin^2 \left( \tan^{-1} \frac{z_4 - z_1}{x_2 - x_1} \right) \right\}^{1/2}
\]

(5d)

If

\[
\frac{\lambda}{\left\{ 1 + (\lambda^2 - 1)\sin^2 \left( \tan^{-1} \frac{z_3 - z_2}{x_2 - x_1} \right) \right\}^{1/2}} = 1
\]

(6a)
and

\[
\lambda \left\{ 1 + \left( \lambda^2 - 1 \right) \sin^2 \left( \tan^{-1} \frac{z_4 - z_1}{x_2 - x_1} \right) \right\}^{1/2} = 1 \quad (6b)
\]

the expressions for \( T_3 \) and \( T_4 \) simplify and Eq. (4) becomes

\[
\Delta V = \frac{I}{4\pi \lambda} \frac{\rho_m}{\rho_H} \left\{ \frac{1}{|z_3 - z_1|} + \frac{1}{|z_4 - z_2|} - \frac{1}{\left( (x_2 - x_1)^2 + (z_3 - z_2)^2 \right)^{1/2}} - \frac{1}{\left( (x_2 - x_1)^2 + (z_4 - z_1)^2 \right)^{1/2}} \right\} \quad (7).
\]

This is exactly the relationship which would be obtained for cross-borehole measurements made in a medium with an isotropic resistivity

\[
\frac{\rho_m}{\lambda} = \frac{\sqrt{\rho_V \rho_H}}{\sqrt{\rho_H}} = \rho_H. \quad (8)
\]

Thus, in such circumstances the cross-borehole measurements will yield the horizontal component \( \rho_H \) of the anisotropic resistivity.

Formally, the conditions under which Eqs. (6a) and (6b) hold are either (i) \( x_2 = x_1 \), or (ii) \( \frac{z_4 - z_1}{x_2 - x_1} \gg 1 \) and \( \frac{z_3 - z_2}{x_2 - x_1} \gg 1 \). The first condition corresponds to that of all the electrodes being in the same borehole. This is essentially the case discussed by Timco (1979) who used four electrodes aligned vertically in the side of an ice pit. Given a finite thickness of sea ice and the impracticality of placing the two boreholes very close together, the second condition cannot be exactly realised. However, for \( |x_2 - x_1| \approx 1 \) m, with electrodes spaced vertically every 10 cm, and for realistic values of \( \lambda \), combinations of current and potential electrodes can be chosen (i.e. values of \( z_1, z_2, z_3 \) and \( z_4 \)) for which both Eqs. (4) and (7) approximate to

\[
\Delta V \approx \frac{I}{4\pi \lambda} \frac{\rho_m}{\rho_H} \left\{ \frac{1}{|z_3 - z_1|} + \frac{1}{|z_4 - z_2|} \right\}. \quad (9)
\]

Using only electrode combinations for which this approximation is valid will therefore allow measurements to be made which are responsive to, and will give a reasonable estimate of, \( \rho_H \).

To illustrate this Table 1 shows typical magnitudes of the four terms \( T_1, T_2, T_3 \) and \( T_4 \) (given by Eqs. (5a)–(5d)) for situations where the anisotropy coefficient \( \lambda \) takes values of 0.1 or 0.3—in line with typical values indicated not only by Fig. 2 but also by other previous surface resistivity soundings (Buckley et al., 1986; Thyssen et al., 1974; Timco, 1979). The results show that, for all of the representative electrode combinations listed, the magnitudes of the first two terms \( (T_1, T_2) \) are significantly larger than those of the third and fourth terms \( (T_3, T_4) \). Eq. (9) may therefore be

<table>
<thead>
<tr>
<th>( z_1 )</th>
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<th>( z_3 )</th>
<th>( z_4 )</th>
<th>( \lambda )</th>
<th>( T_1 )</th>
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<td>0.3</td>
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<tr>
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<tr>
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<td>3.33</td>
<td>0.1</td>
<td>0.08</td>
</tr>
</tbody>
</table>
regarded as a valid approximation to Eq. (4). Note that the smaller the value of the anisotropy coefficient $\lambda$, the less significant are $T_3$ and $T_4$ compared to $T_1$ and $T_2$. For $\lambda < 1$ the factors such as

$$\frac{\lambda}{\left(1 + (\lambda^2 - 1)\sin^2\left(\frac{\tan^{-1}\frac{z_1 - z_2}{y_2 - y_1}}{C_0}\right)\right)^{1/2}}$$

in Eq. (5c) are less than unity. The approximation of Eqs. (7)–(9) is thus less accurate. This places greater constraints on the allowable electrode combinations for which the approximation is valid. As the forward calculation used in the modelling of data is based on Eq. (7) the use of electrode constraints for which the approximation is not good will tend to lead to an overestimate of $\rho_{H}$.

The preceding derivation shows that, unlike resistivity sounding, and with a careful choice of the combinations of electrodes used, cross-borehole resistivity tomography is able to be used to derive one of the principal resistivity values of the anisotropic resistivity structure. Furthermore, assuming that the lowermost electrodes are in the underlying seawater, the vertical alignment of electrodes will also be able to accurately determine the boundary between the relatively resistive sea ice and the highly conducting sea-water.

3. Cross-borehole resistivity measurements

In late January 2006 two vertical strings of electrodes were installed in boreholes, drilled 1 m apart, in approximately 0.8 m thick land-fast first-year sea ice in the Chukchi Sea about 300 m offshore from Pt. Barrow (71° 22′ 03″ N, 156° 31′ 03″ W). The site was also operated as a University of Alaska Fairbanks sea ice mass balance site continuously recording snow and ice thickness, water depth, the air–snow–ice–water temperature profile, air temperature and relative humidity at 2 m above the ice, and developmental sea ice salinity measurements using dielectric probes. The electrode strings (Fig. 3) were constructed at Victoria University of Wellington and utilised 185 cm × 5 cm marine-grade stainless steel washers as electrodes positioned at vertical intervals of 10 cm and held in place by cable glands.

Cross-borehole resistivity measurements were made on three separate occasions as the ice warmed over the period April–June 2006. The first series of measurements was made between 22–25 April at which time the sea ice had thickened to 1.38 m, the second on 10 May (ice thickness 1.46 m), and the final series on 8 June (ice thickness 1.54 m). On each occasion ice cores were taken from the area adjacent to the electrode installation and used to determine the salinity profile within the ice. Each set of resistivity measurements comprised approximately 500 individual readings, using one current and

![Fig. 3. Details of the construction of the vertical electrode strings.](image)
Fig. 4. (a): Horizontal slices through the 3D resistivity model derived from cross-borehole resistivity measurements made between 22–25 April 2006. (b): Horizontal slices through the 3D resistivity model derived from cross-borehole resistivity measurements made on 11 May 2006. (c): Horizontal slices through the 3D resistivity model derived from cross-borehole resistivity measurements made on 8 June 2006.
one potential electrode in each borehole, and with the choice of electrodes constrained to satisfy the conditions for Eq. (9) to be a valid approximation to both Eqs. (4) and (7). For each set of measurements the entire data set was inverted, using the Res2Dinv™ and Res3Dinv™ codes, to derive both 2 and 3-dimensional models of the

Fig. 4 (continued).
sea ice resistivity structure between the boreholes. The results of the 3D inversions of the three separate sets of measurements are presented in Fig. 4, which shows horizontal slices through the derived 3D models, and Fig. 5, which shows a vertical cross-section through each model and also the measured temperature and salinity profiles. In Fig. 4 the boreholes are located at the top left \((x=-0.4\ m, y=-0.3\ m)\) and bottom right \((x=0.4\ m, y=0.3\ m)\) of each slice. Fig. 5 shows vertical sections along the line \(y=0\ m\).
It is clear from Fig. 4, and from the result of the 2D inversions which are not shown, that a halo of anomalous resistivity structure is caused by the presence of the electrode strings. This suggests that as the ice reforms around the electrode strings after their insertion, and subsequently thickens, the microstructure in the immediate vicinity of the strings is significantly affected by their presence. The size of the affected region will depend upon the dimensions of the holes drilled for insertion of the electrode strings and on the dimensions of the electrodes themselves. The apparent size of the affected region in the derived models will also depend upon the dimensions of the grid cells defined for the inversion. As is discussed below, away from the immediate vicinity of the electrode strings the derived resistivity structure and its development over time is inherently reasonable. The existence of the halo around the electrode strings provides a strong argument therefore why it is preferable to use multiple boreholes to determine $\rho_{H}$ rather than single borehole measurements. It can also be noted from Fig. 5 that the underlying sea-water is clearly resolved beneath the sea ice as an effective half space with a resistivity of approximately $0.4 \, \Omega\,m$.

Figs. 4 and 5 show that the resistivity structure of the sea ice varies both spatially and temporally. To aid in an understanding of the structure the measured temperature and salinity profiles have been used to calculate the brine volume percentage using the relationships proposed by Cox and Weeks (1983). The resulting brine volume fractions for each of the three sets of measurements are also shown in Fig. 5. Note that for the final set of measurements (Fig. 5c) low salinity values over the top 40 cm are due to the penetration of fresh meltwater from surface snow. The calculated brine volume fraction $V_b$ is here least reliable due to high sensitivity to measurement uncertainties in both salinity and temperature.

Considering Fig. 5, the following observations can be made.

1. For the first set of measurements (22–25 April, Fig. 5a) $\rho_{H}$ is high ($500–5000 \, \Omega\,m$) from the surface of the ice down to about 1.25 m depth. Maximum $\rho_{H}$ occurs at about 0.9 m above which ice temperature is less than $-5 ^\circ C$, and the brine volume <5%.
2. Near the base of the ice (1.25–1.38 m) the calculated brine volume fraction rises rapidly to about 15% and the resistivity decreases sharply to $<10 \, \Omega\,m$. In this depth range the temperature of the ice is above $-5 ^\circ C$ and salinity rises to >6%.
3. The measurements made on 11 May (Fig. 5b) show that a lower resistivity region ($\sim 200 \, \Omega\,m$) has developed between the surface and 0.45 m depth. Compared with the first measurements, the salinities in this region remain around 7‰, but with temperatures increased to just above $-5 ^\circ C$, the brine volume has increased to about 6%.
4. Below this lower resistivity layer and down to about 1 m depth, brine volume fractions are between about 4 and 7% and correlate with significantly higher resistivities ($\geq 500 \, \Omega\,m$). The temperature in this depth range is close to $-5 ^\circ C$ and the salinity is about 4‰.
5. Below 1 m in depth the temperature, salinity and brine volume percentage all increase: to about $-2 ^\circ C$, >7‰, and >15‰, respectively. The resistivity decreases to around $2 \, \Omega\,m$ at the base of the sea ice.
6. On 8 June (Fig. 5c), there is a clear correlation in the upper 0.5 m between low salinities, resulting from penetration of meltwater, and resistivity values of $\sim 30 \, \Omega\,m$. At these high temperatures (T$\geq -2 ^\circ C$), brine volumes are very sensitive to salinity which can be underestimated due to core drainage (see below also). This correlation of low resistivity with low salinity and brine volume fraction in the near surface is somewhat surprising and requires further investigation.
7. Resistivity is also lowered throughout the rest of the ice with brine volume fractions $\geq 10%$. However, uncertainties in the salinity profile mean that variations in this deeper part of the sea ice are not well resolved.

These observations clearly lead to the general inference that a decrease in the horizontal component of the bulk resistivity is associated with an increase in brine volume fraction. Significant decreases in resistivity appear to occur when the brine volume fraction reaches somewhere between 5 and 7%. As the resistivity of the brine component is much less than that of the solid ice component such a general observation makes sense. The variation of bulk resistivity in such a two component system where the resistivity of one component is much higher than that of the other, has traditionally been
expressed in terms of Archie’s Law (Archie, 1942). For a situation in which all pore spaces are saturated, this may be expressed as

\[
\rho = \phi^{-m} \rho_b
\]

where \(\phi\) is the porosity, \(m\) is an empirically determined parameter, and \(\rho_b\) is the resistivity of the more conductive phase. In the present situation \(\rho_b\) therefore represents the resistivity of the brine. For sea ice brine inclusions, the brine inclusion salinity \(S_b\) is a function of only temperature and \(\rho_b\) can also be calculated as a function of only temperature. Morey et al. (1984) give equations to calculate \(\rho_b(T)\) up to \(-2.0\) °C. We have extended this approach to \(0\) °C using the \(S_b(T)\) relationship found in Lepparanta and Manninen (1988). Using these \(\rho_b(T)\) values we follow the approach of Morey et al. (1984) and analyze our results in terms of the ‘formation factor’ \(FF = \rho/\rho_b\), where \(\rho\) here is the measured value of \(\rho_H\), and according to Archie’s Law, \(FF = \phi^{-m}\).

Fig. 6 shows the formation factor as a function of porosity for our three sets of measurements (solid symbols) and also the previous measurements of Morey et al. (1984) and our calculations of FF from the measurements of Thyssen et al. (1974) and Buckley et al. (1986). All measurements are for FY ice, and \(V_b(S,T)\) and \(\rho_b(T)\) were calculated identically for all data. Therefore larger FF values indicate higher measured values of resistivity. Lines indicate Archie’s Law behaviour, \(FF = \phi^{-m}\), for different \(m\) values.

Our data are seen to depart from Archie’s Law behavior above a brine volume fraction of about \(V_b^* \approx 8\%\), although the precise point of departure is not well constrained. As discussed below this probably indicates the effect of enhanced connectivity between brine pockets at high brine volume fractions. We note that the \(v_b\) values from June 8 (stars) are somewhat sensitive to the uncertainty in salinity measurements, but with little effect on our overall conclusions. For such high temperatures \((T>-2.5\) °C) \(v_b\) is increasingly sensitive to salinity, which can be underestimated through brine drainage during coring. For a maximum salinity underestimate of 20%, \(V_b\) values (in units of %) for these points are underestimated by 4.5±2.2, but with only a slight increase in

Fig. 6. Log–log plot of formation factor (FF) as a function of brine volume fraction. Solid symbols are our measurements. Squares − from 22–25 April; triangles − 11 May; stars − 8 June 2006. Open symbols are from the three sets of measurements of Morey et al. (1984) at two different sites. Crosses are horizontal resistivities from Timco (1979). Asterisks are layer averages from the three measurements of Buckley et al. (1986) at two sites in McMurdo Sound. All measurements were made in FY ice.
$V_b^{*} \approx 9-10\%$. These measurements overlap with the results from Thyssen et al. (1974) and Buckley et al. (1986), but are significantly higher than those of Morey et al. (1984). The reason for this discrepancy is not known, but we consider it unlikely that our measurements have significantly overestimated the resistivity. For $V_b < 10\%$, a regression on Eq: (8) gives a best-fit value of $m=2.88$. Some care should be taken in interpreting this value however, as relaxing the constraint of FF = 1 at $V_b = 100\%$ leads to higher $m$ values.

### 4. Discussion and conclusions

Numerous observations suggest that the brine component of sea ice undergoes a percolation transition, such that upon warming the brine pockets in cold ice become interconnected over larger scales as the temperature increases. At this point the bulk physical properties of sea ice become dominated by the physical properties of the brine component. Such a transition is common in many 2-phase systems. Within sea ice it has been suggested that the percolation transition occurs at a brine volume fraction of about 5\% (Golden et al., 1998; Golden, 2000, 2003) and as the volume fraction of brine is a function of temperature, the transition is expected to be approached in a continuous manner (Light et al., 2003). In sea ice with a typical salinity of 5–6 psu this should occur at a temperature of about $-5^\circ C$.

The derived behaviour of the horizontal component of the bulk resistivity is entirely consistent with these expectations. Through the bulk of the sea ice there is a general decrease in resistivity with increasing salinity and brine volume fraction. The analysis in section 3 suggests that the percolation transition occurs at a brine volume fraction of about 5\% (Golden et al., 1998; Golden, 2000, 2003) and as the volume fraction of brine is a function of temperature, the transition is expected to be approached in a continuous manner (Light et al., 2003). In sea ice with a typical salinity of 5–6 psu this should occur at a temperature of about $-5^\circ C$.

We have demonstrated that cross-borehole resistivity tomography is able to resolve the spatial variation in the horizontal component of the anisotropic resistivity of sea ice. Repeat measurements made over the period during which the ice warmed during spring have shown temporal variations in $\rho_{H}$ which correlate with changes in the brine volume fraction and, at lower temperatures and brine volume fractions, are broadly consistent with Archie’s Law. At temperatures above $-5^\circ C$ there is evidence of a percolation transition. Further measurements planned using 4 electrode strings are aimed at producing improved resolution of the resistivity structure and its variation with time. Improved resolution will also be aided by redesign of the electrode strings to reduce the size of the “halo” region around the strings, and by use of smaller and more closely spaced electrodes. Correlation of cross-borehole measurements with surface resistivity soundings may also allow more complete information to be extracted concerning the spatial and temporal evolution of the anisotropic resistivity structure as a whole, and its relation to the microstructure of sea ice.

### Acknowledgements

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### Appendix A. Electric current and potential in an anisotropic medium (Bhattacharya and Patra, 1968)

In an anisotropic medium in which there is an electric current the current density is related to electric field by

\[
J_x = \sigma_{xx}E_x + \sigma_{xy}E_y + \sigma_{xz}E_z
\]

\[
J_y = \sigma_{yx}E_x + \sigma_{yy}E_y + \sigma_{yz}E_z
\]

\[
J_z = \sigma_{zx}E_x + \sigma_{zy}E_y + \sigma_{zz}E_z
\]

where it can be demonstrated that $\sigma_{jk}=\sigma_{kj}$.

When the axial system is oriented parallel to the principal axes of anisotropy (A1) reduces to

\[
J_x = \sigma_{xx}E_x = \frac{1}{\rho_x}E_x
\]

\[
J_y = \sigma_{yy}E_y = \frac{1}{\rho_y}E_y
\]

\[
J_z = \sigma_{zz}E_z = \frac{1}{\rho_z}E_z
\]

where the $\rho$’s are now the principal resistivities.
Continuity of electric current (\(\text{div } J=0\)) now gives

\[
\frac{\partial}{\partial x} \left( \frac{E_x}{\rho_x} \right) + \frac{\partial}{\partial y} \left( \frac{E_y}{\rho_y} \right) + \frac{\partial}{\partial z} \left( \frac{E_z}{\rho_z} \right) = 0 \tag{A3}
\]

and if the medium is homogeneously anisotropic, then expressing the electric field as the gradient of a scalar potential \(V\) reduces (A3) to

\[
\frac{1}{\rho_x} \frac{\partial^2 V}{\partial x^2} + \frac{1}{\rho_y} \frac{\partial^2 V}{\partial y^2} + \frac{1}{\rho_z} \frac{\partial^2 V}{\partial z^2} = 0. \tag{A4}
\]

If we now redefine the co-ordinate system by

\[
\alpha = x\sqrt{\rho_x}, \quad \beta = y\sqrt{\rho_y}, \quad \gamma = z\sqrt{\rho_z}
\]

then Eq. (A4) reduces to Laplace’s equation

\[
\frac{\partial^2 V}{\partial \alpha^2} + \frac{\partial^2 V}{\partial \beta^2} + \frac{\partial^2 V}{\partial \gamma^2} = 0,
\]

which has a solution

\[
V = \frac{C}{(\alpha^2 + \beta^2 + \gamma^2)^{1/2}} = \frac{C}{(\rho_{x}\alpha^2 + \rho_{y}\beta^2 + \rho_{z}\gamma^2)^{1/2}} \tag{A5}
\]

where \(C\) is a constant of integration.

The equipotential surfaces, defined by,

\[
\rho_{x}\alpha^2 + \rho_{y}\beta^2 + \rho_{z}\gamma^2 = K^2
\]

are ellipsoids with axes coinciding with the principal axes of anisotropy. The current densities are given by:

\[
J_x = -\frac{1}{\rho_x} \frac{\partial V}{\partial \alpha} = \frac{Cx}{(\rho_{x}\alpha^2 + \rho_{y}\beta^2 + \rho_{z}\gamma^2)^{3/2}}
\]

\[
J_y = -\frac{1}{\rho_y} \frac{\partial V}{\partial \beta} = \frac{Cy}{(\rho_{x}\alpha^2 + \rho_{y}\beta^2 + \rho_{z}\gamma^2)^{3/2}}
\]

\[
J_z = -\frac{1}{\rho_z} \frac{\partial V}{\partial \gamma} = \frac{Cz}{(\rho_{x}\alpha^2 + \rho_{y}\beta^2 + \rho_{z}\gamma^2)^{3/2}} \tag{A6}
\]

which shows that

\[
\frac{J_x}{\alpha} = \frac{J_y}{\beta} = \frac{J_z}{\gamma}
\]

This implies that current lines spread out radially from the source as they do in an isotropic medium. The electric field only coincides with the current lines along the principal axes.

In most geological situations bedding exists. In the bedding plane the resistivity is generally isotropic and we refer to it as \(\rho_S\). Perpendicular to the bedding the transverse resistivity is \(\rho_T\). In the case of sea ice, with brine inclusions oriented vertically, the bedding plane may be considered to be horizontal and we identify \(\rho_S\) with the horizontal resistivity and \(\rho_T\) with the vertical resistivity. For a horizontal bedding Eq. (A4) becomes

\[
\frac{1}{\rho_S} \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + \frac{1}{\rho_T} \frac{\partial^2 V}{\partial z^2} = 0 \tag{A7}
\]

and the equipotential surfaces

\[
x^2 + y^2 + \frac{\rho_T}{\rho_S} z^2 = K^2
\]

are ellipsoids of revolution about the vertical axis.

If we now define the coefficient of anisotropy \(\lambda\) and the mean resistivity \(\rho_m\) as

\[
\lambda = \sqrt{\frac{\rho_T}{\rho_S}} \quad \rho_m = \sqrt{\rho_T\rho_S}
\]

the solution for \(V\) (A5) can be written as

\[
V = \frac{C}{\rho_m^{1/2}(x^2 + y^2 + \lambda^2 z^2)^{1/2}} \tag{A8}
\]

and the current densities as

\[
J_x = \frac{Cx}{\rho_m^{3/2}(x^2 + y^2 + \lambda^2 z^2)^{3/2}}
\]

\[
J_y = \frac{Cy}{\rho_m^{3/2}(x^2 + y^2 + \lambda^2 z^2)^{3/2}}
\]

\[
J_z = \frac{Cz}{\rho_m^{3/2}(x^2 + y^2 + \lambda^2 z^2)^{3/2}}
\]

The total current density is

\[
J = \frac{C(x^2 + y^2 + z^2)^{1/2}}{\rho_m^{3/2}(x^2 + y^2 + \lambda^2 z^2)^{3/2}} \tag{A9}
\]

Suppose now that a current \(I\) is passed into the ground through a single electrode embedded in the anisotropic medium. The current must subsequently pass through a sphere of radius \(R\) centred on the electrode. Namely

\[
I = \int J \, ds = \int_0^{2\pi} d\theta \int_0^\pi J \, R^2 \sin \theta \, d\theta \, d\phi
\]

where \(\theta\) and \(\phi\) are the polar and azimuthal angles, respectively.
As $x^2 + y^2 = R^2 \sin^2 \theta$ and $z^2 = R^2 \cos^2 \theta$, $J$ can be written as
\[ J = \frac{C}{\rho_S^{3/2}} R^2 (\sin^2 \theta + \lambda^2 \cos^2 \theta)^{3/2} \]
and
\[ I = \frac{C}{\rho_S^{3/2}} \int_0^{2\pi} \sin \theta d\theta \int_0^{\pi} \left\{ 1 + (\lambda^2 - 1) \cos^2 \theta \right\}^{3/2} = \frac{4\pi C}{\lambda \rho_S^{3/2}} \]
which gives the constant $C$ as
\[ C = \frac{I}{4\pi \lambda \rho_S^{3/2}} \quad (A10) \]
This means that (A8) may be written in the form that the potential at a location $(R, \theta)$ from a single electrode is given by
\[ V = \frac{I \rho_m}{4\pi R \{ 1 + (\lambda^2 - 1) \cos^2 \theta \}^{1/2}}. \quad (A11a) \]
In the situation where the current electrode is not within the anisotropic medium but on the surface of it, the limits to the integration over $\theta$ are $\pi/2$ and $\pi$ and the result is
\[ V = \frac{I \rho_m}{2\pi R \{ 1 + (\lambda^2 - 1) \cos^2 \theta \}^{1/2}} \quad (A11b) \]

References


Beckmann, A., Goosse, H., 2003. A parameterization of ice shelf which gives the constant $\lambda$ and $C$.


